Effects of Beam Velocity Spread on Two-Stage Tapered Gyro-TWT Amplifier

Khanh T. Nguyen, Gun-Sik Park, Jin Joo Choi, Soo-Yong Park, and Robert K. Parker, Fellow, IEEE

Abstract—The recent successful operation of a two-stage Kα-band gyro-TWT amplifier at the Naval Research Laboratory has demonstrated its viability as a source of broadband high-power millimeter waves. However, it has been observed experimentally that sharp gain reductions occur at certain frequencies. In this paper, an analytical theory is presented which describes the effect of electron beam quality on the device's instantaneous bandwidth. It is shown that these observed gain dips are due to electron phase-mixing, resulting from finite beam velocity spreads, in the frequency-dependent sever. In addition, the theory demonstrates that the velocity-spread induced dips can be shifted or reduced by adjusting the magnetic field profile, circuit length, and beam axial velocity, consistent with experimental observations.

STABLE operation at a saturated gain of 25 dB over a 20% instantaneous bandwidth has been observed with a two-stage Kα-band gyrotrotron travelling wave tube (gyro-TWT) amplifier at the Naval Research Laboratory [1]. A saturated efficiency of 16% was obtained with a 33 kV, 1.5 A electron beam. The device incorporates precise axial tapering of both the magnetic field profile and the TE_{10} rectangular interaction circuit to obtain the measured high performance. The objective of this experimental program is to demonstrate the viability of gyro-TWT's [2], [3] as a promising source of broadband, high-power millimeter waves at relatively low beam voltage. Applications for such devices include radar, electronic warfare, and communication systems.

Previously, a bandwidth of more than 30% has been achieved in a single stage tapered Ka-band gyro-TWT [4]. However, as expected for a single port reflection-type amplifier, the high gain operation of the tube was limited by multi-pass interference effects and tube operation more than 20 dB resulted in excessive gain fluctuations across the amplification band. These deficiencies are overcome with a two-stage circuit, in which the gain per stage is reduced less than 1%. This requirement on electron beam quality can be satisfied by the new generation of electron guns, currently under development [7]. However, substantial velocity spreads are to be expected with most existing guns and, in particular, the double-anode magnetron injection gun (MIG) employed in these experiments.

Since the sever length is frequency-dependent in the tapered gyro-TWT, it is reasonable to expect the beam velocity spread to play a critical role in determining the device's instantaneous bandwidth. That is, the combination of velocity spread and tapered geometry can cause the phase modulation impressed by the RF on the electrons in the first stage interaction to add constructively or destructively at the second stage interaction, depending on the sever length at that frequency. This is a result of the electrons with different axial velocities drifting through the frequency-dependent length of the sever and the nonuniform magnetic field. Sharp drop in gain can occur at frequencies where the electrons add up destructively in phase. The impact of beam axial velocity spread on the instantaneous bandwidth of the two-stage tapered gyro-TWT can be...
understood as follows. The initial perpendicular momentum of electrons upon arriving at the first stage interaction region, can be expressed as

$$P_{\perp}(z = z_1) = P_{\perp 0}(z_1) e^{-i \phi_0}$$  \hspace{1cm} (1)$$

where $z_1(f_0)$ is the location of the first stage interaction for frequency $f_0$, $\phi_0$ is the electron's initial phase and is assumed to be uniformly distributed between 0 and $2\pi$, and $P_{\perp 0}$ is the magnitude of the initial perpendicular momentum at $z_1$. The beam-wave interaction at $z_1$ results in a modulation of the electron energy, the magnitude of which depends on the relative phase between the electrons and the RF wave. A detailed analysis of this interaction has been performed by Ganguly and Ahn [6], and we refer readers to their publication for further details. For the problem at hand, it is sufficient to assume that the interaction region for each frequency in each stage is short and that the resulting energy modulation in the first stage can be expressed in terms of the relativistic factor as

$$\gamma = \gamma_0 + \Delta \gamma_m \cos(\phi_0 + \phi_1)$$  \hspace{1cm} (2)$$

where $\gamma_0 = (1 - v_0^2/c^2)^{-1/2}$ is the initial relativistic factor of the electrons, $\Delta \gamma_m \ll (\gamma_0 - 1)$ is the maximum magnitude of the energy modulation, and $\phi_1 = 2\pi f_0 t_1 = \omega_0 t_1$ is the instantaneous phase of the RF wave at $z_1$ and $t_1$.

Following the first-stage interaction region, the electron perpendicular momentum at any position in the drift (sever) region, $z = z_1 + \int_{t_1}^{t^*} v_z dt'$, can then be written approximately as

$$P_{\perp}(z) \approx P_{\perp 0}(z) \exp \left[ -i \int_{z_1}^{z} \Omega(z') dt' \right]$$  \hspace{1cm} (3)$$

where $\Omega(z') = eB(z')/(\gamma_0 mc)$ is the electron cyclotron frequency at $z' = z(t')$. The above expression can also be written in the following form:

$$P_{\perp}(z) \approx P_{\perp 0}(z) \exp \left[ -i \phi_0 - \int_{z_1}^{z} \Omega(z') dt' \right]$$  \hspace{1cm} (4)$$

where we have employed the modulated relativistic factor, and defined $\Omega_0(z') = eB(z')/(\gamma_0 mc)$ as the unperturbed cyclotron frequency.

From the expression for the perpendicular momentum, we can readily calculate the fundamental component of the perturbed current for angular frequency, $\omega_0$ at $z$. This is given in terms of the beam total current, $I_0$, as

$$I_p(\omega_0, z) = \frac{I_0}{2 \pi} \int_{0}^{2\pi} d\phi_0 \int_{-\infty}^{\infty} dv_z f(v_z) \left[ \frac{P_{\perp 0}(z)}{P_{\perp 0}(z)} e^{-i\omega_0 t_1} \right]$$

\hspace{1cm} = I_0 \int_{-\infty}^{\infty} dv_z f(v_z) \left[ \frac{P_{\perp 0}(z)}{P_{\perp 0}(z)} \right] J_1(\gamma_0) \cdot \exp \left[ -i \left( \omega_0(t - t_1) - \int_{z_1}^{z} \Omega_0(z') \frac{dz'}{v_z} \right) \right]$$

$$= I_0 \int_{-\infty}^{\infty} dv_z f(v_z) \left[ \frac{P_{\perp 0}(z)}{P_{\perp 0}(z)} \right] J_1(\gamma_0) \cdot \exp \left[ -i \left( \omega_0(t - t_1) - \int_{z_1}^{z} \Omega_0(z') \frac{dz'}{v_z} \right) \right]$$

(5)

Here, $f(v_z)$ is the axial velocity distribution function, and we have performed the integral over the initial phase, $\phi_0$. The argument of the first-order Bessel function, $J_1(\gamma_0)$, has been defined as the phase bunching parameter,

$$X(z, z_1) = \frac{\Delta \gamma_m}{\gamma_0} \int_{z_1(\omega_0)}^{\infty} \Omega_0(z') \frac{dz'}{v_z(z')}$$

(6)

Note that the phase bunching parameter increases as a function of $z$ in the sever region. This is a consequence of the usual ballistic bunching in two stage gyro-TWT's. For certain parameter regimes, overbunching can occur which may result in $J_1(\gamma_0) \approx 0$ at the second-stage interaction location, $z_2(\omega_0)$. However, using the parameters employed in these experiments, simulations with grazing magnetic field profile and with no axial velocity spread, i.e.,

$$f(v_z) = \delta(v_z - v_{z0})$$

(7)

have indicated that uniform saturated gain across the entire frequency band is indeed feasible. Thus, by defining the fundamental component of the perturbed current in the case of no velocity spread at $z_2(\omega_0)$ as $I_{p0}(\omega_0)$, we can then write the perturbed current for beam with finite velocity spread, $\Delta v_z \neq 0$, using (5), approximately as

$$I_p(\omega_0, \Delta v_z) = I_{p0}(\omega_0) S(\omega_0, \Delta v_z)$$

(8)

Here

$$I_{p0}(\omega_0) = \frac{I_0 P_{\perp 0}}{P_{\perp 0}} J_1[X(v_z, z_1) - \int_{z_1(\omega_0)}^{\infty} \Omega_0(z') \frac{dz'}{v_z(z')}]$$

and

$$S(\omega_0, \Delta v_z) = \int_{-\infty}^{\infty} dv_z f(v_z) \left[ \frac{P_{\perp 0}(z)}{P_{\perp 0}(z)} \right] J_1(\gamma_0) \cdot \exp \left[ -i \int_{z_1}^{z} \Omega_0(z') \frac{dz'}{v_z(z')} \right]$$

(9)

is defined as the normalized spectral amplitude due to beam axial velocity spread. It should be noted that $I_p$ is essentially the input signal which will drive the beam-wave interaction in the second stage. Obviously, it is desirable to have $S(\omega_0, \Delta v_z) = 1$ across the whole bandwidth. This is readily satisfied in the case of $\Delta v_z = 0$ (no velocity spread), or in the case of when

$$\Delta v_z \leq 2\pi \frac{\omega_0}{v_{z0}}$$

such as in a relatively short sever of a two-stage, uniform, gyro-TWT. In the two-stage tapered gyro-TWT, $[\omega_0 - \Omega_0(z)]/v_{z0}(z)$ which is the pitch angle of the electron bunch at frequency $\omega_0$, can vary substantially in the frequency-dependent sever region. This phenomenon is illustrated in Fig. 1, which shows the spatial phase space of the electrons from a MAGIC [8] simulation of the two-stage tapered gyro-TWT at 32 GHz with no velocity spread and grazing magnetic field
The total gain for both stages from this simulation is 26 dB in good agreement with the maximum gain from experiments. The transitions of the electron bunch pitch angle from positive \((\omega_0 > \Omega_0(z))\) at the first-stage interaction region, to negative \((\omega_0 < \Omega_0(z))\) in the sever region, and then back to positive again just before the interaction region in the second stage can be clearly seen in Fig. 1. It is this type of behavior that, when coupled with beam velocity spread, can produce sharp dips in the instantaneous bandwidth.

To quantify the impact of beam velocity spreads on the instantaneous bandwidth, we assume a simple axial velocity spread distribution function of the form

\[ f(v_z) = \begin{cases} \frac{1}{2\sqrt{3}\Delta v_z}, & \text{for } -\sqrt{3}\Delta v_z < v_z - v_{z0} < \sqrt{3}\Delta v_z, \\ 0, & \text{otherwise} \end{cases} \]

(9)

where \(\Delta v_z\) is the root-mean-square axial beam velocity spread. With this flat-top distribution function, the normalized spectral amplitude can easily be shown to be

\[ S(\omega_0, \Delta v_z) = \frac{\sin(\Delta \varphi)}{\Delta \varphi}. \]

(10)

Here, \(\Delta \varphi\) is defined as the phase spread and is given by

\[ \Delta \varphi = \sqrt{3} \int_{z_1(\omega_0)}^{z_2(\omega_0)} \left[ \omega_0 - \Omega_0(z') \right] \frac{\Delta v_z(z')}{v_{z0}(z')^2} dz'. \]

For the sake of completeness, an equivalent expression has also been derived for a gaussian distribution function (see Appendix.) The use of a different distribution function will not, however, qualitatively change the physical characteristics described in the following discussion.

It should be noted that since the magnetic field is tapered, both the beam axial velocity and velocity spread are functions of \(z\). These are given by

\[ v_{z0}(z) = v_{z0}(0) \cdot \left[ 1 + \alpha_0^2 \left( 1 - \frac{B_z(z)}{B_z(0)} \right)^{1/2} \right] \]

(11)

and

\[ \frac{\Delta v_z(z)}{v_{z0}(z)} = \frac{\Delta v_z(0)}{v_{z0}(0)} \left[ 1 + \alpha_0^2 \left( 1 - \frac{B_z(z)}{B_z(0)} \right)^{-1} \right]^{-1}. \]

(12)

Here \(z = 0\) is the start of the first stage and \(\alpha_0\) is the initial velocity ratio.

In the absence of velocity spread, (10) shows that \(S(\omega_0, 0) = 1\) for all frequencies. For finite velocity spread, \(\Delta \varphi\) must be evaluated for each frequency. However, several observations can readily be made from inspection of (10). First, velocity spread has very little impact for frequencies near the high end of the frequency band, since the sever length is relatively short. Second, for large velocity spreads, the first dip will most likely occur at frequency \(\omega_1 = \Omega_{\text{max}}\), where \(\Omega_{\text{max}}\) is the maximum cyclotron frequency. At this frequency, \(\Delta \varphi\) is positive and is the maximum for the entire frequency band. Third, for a monotonically tapered magnetic field, there will always be a second passband without attenuation centered around frequency \(\omega_2\), where \(\Delta \varphi\) is identically zero. This is because the interaction occurs at \(z_1\) and \(z_2\) where \(\omega_2 > \Omega_0(z_1) = \Omega_0(z_2)\) and electrons pass through the sever where \(\omega_2 < \Omega_{\text{max}}\). Fourth, for \(\omega_0 < \omega_2\), \(\Delta \varphi\) will quickly turn very negative, and substantial attenuation can be expected for large velocity spread. This is simply due to the fact that, at these frequencies, electrons spend a substantial amount of time in the long sever where \(\omega_0 < \Omega_0(z)\). Thus, the phase modulation impressed on the electrons in the first stage is substantially phase mixed before reaching the second stage interaction location \(z_2\). Finally, larger beam velocity spreads increase the sensitivity of the instantaneous bandwidth to the magnetic field profile.

Results from the numerical integration of (10), are shown in Figs. 2, 4, and 5, for the present experimental parameters. It is important to emphasize that the results shown here are to illustrate and quantify the impact of beam velocity spread on the instantaneous bandwidth of the two-stage tapered gyro-TWT only. Nonlinear beam-wave interactions in the first and second stages may introduce other effects on amplifier frequency response which are beyond the scope of this paper, and will not be discussed in detail here. In these figures, the
beam energy and initial velocity ratio $\alpha_0$ are 33 keV and 0.8, respectively. Fig. 2 shows the phase spread, $\Delta \varphi$, the normalized spectral amplitude, $S$, and the attenuation factor, $A = 20 \log_2 (S)$ (in dB), for the case when the axial velocity spread, $\Delta v_x/v_x$, is 7%. In this figure, a deviation from the grazing magnetic field profile has been used. This magnetic field profile results in a grazing intersection between the beam and waveguide mode at the low end of the frequency band and below grazing at the high end. It is interesting to note the strong dip slightly below 36 GHz; the second pass-band centering around 32 GHz and the rapid attenuation due to phase-mixing for frequencies below 32 GHz in Fig. 2.

The frequency response behavior as described above has been observed experimentally as shown in Fig. 3, and provides the impetus for the present work. It can, however, be noted that the dip in the measured bandwidth appears to be sharper than that of the theoretical curve. We believe this is due in part to the fact that the attenuation factor, $A$, only represents the filtering effects resulting from spread-induced electron phase-mixing in the frequency-dependent sever. It does not include the impact of the saturated gain due to nonlinear beam-wave interactions in the second stage as is the case for the experimental data shown in Fig. 3. Including the effect of saturated gain will tend to flatten the gain near the passbands (edges of dip); hence, increasing the sharpness of the dip. In addition, if the beam spread follows more closely to that of a gaussian (see Appendix) rather than the flat top as shown, the peak attenuation of the dip will be much larger than that presented here. For illustrative purposes, using same parameters as in Fig. 2, the attenuation at the nadir will be $-50$ dB for the gaussian distribution function versus the $A = -24$ dB shown for the flat top. At this level of attenuation, the dip will be quite sharp even with the higher gain of the small-signal regime. Indeed, severe attenuation due to phase-mixing in the sever can even result in negative overall gain. This is seen in Fig. 3 at the dip nadir and also at the low frequency end as expected theoretically.

The effects of velocity spreads of 2, 4, and 7% are shown in Fig. 4(a)–(c), respectively, for the same magnetic field profile of Fig. 2. Note the merging of the two pass-bands and improved bandwidth as the velocity spread is reduced. Similar behavior can also be achieved by reducing the circuit length and/or increasing beam axial velocity.

The sensitive dependence of the instantaneous bandwidth on the magnetic field profile, for a fixed 7% velocity spread, is shown in Fig. 5. The magnetic field profile for Fig. 5(a) is the same as for Fig. 2. A grazing magnetic field profile is employed in Fig. 5(b). In Fig. 5(c), the magnetic field is linearly varied from below grazing at low frequencies to grazing at the high end. The bandwidth for this case is 20%, and is about the same as the best bandwidth achieved experimentally by profiling the magnetic field as shown in Fig. 6.

To extend the bandwidth beyond 20% with the beam quality available with the present experimental gun, one could 1) reduce the circuit length (at the expense of gain), 2) increase the beam axial velocity by reducing $\alpha_0$ (at the expense of efficiency), or 3) increase the beam voltage. Combinations
of these techniques would also extend the bandwidth. Reducing the beam velocity ratio also has an added effect of reducing the beam axial velocity spread, since \( \Delta v_z/v_z = \alpha^2 \Delta v_{\perp}/v_{\perp} \), and \( \Delta v_{\perp}/v_{\perp} \) remains relative constant with respect to \( \alpha \).

In conclusion, we have presented here an analysis of the effects of beam velocity spread on the instantaneous bandwidth of the two-stage tapered gyro-TWT amplifier. An analytical expression has been derived which shows the dependence of bandwidth on the magnetic field profile, axial velocity, velocity spread, and circuit length. Theoretical results are consistent with experimental observations.

**APPENDIX**

**PHASE-MIXING FOR BEAMS WITH GAUSSIAN SPREAD**

The choice for the electron beam axial velocity distribution function is often the result of a judicious determination of the main cause for beam spread in the beam forming system. For systems where finite cathode width (i.e., finite canonical angular momentum spread) is the main cause of the spread, a flat-top distribution function is a reasonable assumption. On the other hand, a gaussian distribution function is perhaps more realistic, if cathode thermal effects and/or surface roughness are the dominant cause. Thus, in this appendix, the case for beams with gaussian spread, i.e.,

\[
\tilde{f}(v_z) = \frac{1}{\sqrt{2\pi} \Delta v_z} \exp \left\{ -\frac{1}{2} \left( \frac{v_z - v_{z0}}{\Delta v_z} \right)^2 \right\} \quad \text{(A1)}
\]

is presented and briefly discussed.

Substituting (A1) into the expression for the normalized spectral amplitude (8)

\[
S(\omega_0, \Delta v_z) = \int_{-\infty}^{\infty} dv_z f(v_z) \exp \left\{ -i \int_{z_1}^{z_2} \left[ \omega_0 - \Omega_0(z') \right] \frac{v_z - v_{z0}}{v_{z0}} \, dz' \right\}
\]

and performing the integration over \( v_z \), we obtain the normalized spectral amplitude for gaussian beams,

\[
S(\omega_0, \Delta v_z) = \exp \left( -\frac{3}{4} \Delta \varphi^2 \right) \quad \text{(A2)}
\]

Here, we have kept the same definition of \( \Delta \varphi \), the phase spread, as in (10),

\[
\Delta \varphi = \sqrt{3} \int_{z_1(\omega_0)}^{z_2(\omega_0)} \left[ \omega_0 - \Omega_0(z') \right] \frac{\Delta v_z(z')}{v_{z0}(z')^2} \, dz'.
\]

With this notation, the expression for gaussian beam spread [(A2)] can be readily compared against the expression for flat-top beam spread [(10)],

\[
S(\omega_0, \Delta v_z) = \frac{\sin (\Delta \varphi)}{\Delta \varphi}
\]

It can be observed that in the absence of velocity spread, \( \Delta v_z/v_z = 0 \) (hence, \( \Delta \varphi = 0 \)), \( S(\omega_0, \Delta v_z = 0) = 1.0 \) across the whole bandwidth in both expressions. More importantly, in the presence of beam spread, the frequencies where passbands and dip occur do not change with the use of a different distribution function, since those are determined by \( \Delta \varphi (\Delta \varphi = 0 \text{ for passbands, and } \Delta \varphi \text{ is at a local extrema for dip.}) \) The primary difference between these cases is in the level of attenuation for...
a given value of $\Delta \varphi$. It can be expected that the attenuation at the dip nadir will be much more severe for gaussian beams due to the exponential nature of (A2).

ACKNOWLEDGMENT

The authors would like to acknowledge useful discussions with Dr. C. M. Armstrong. Special thanks are also due to Dr. A. K. Ganguly, Prof. Y. Y. Lau, and Dr. D. Pershing for a review of this work. This work is supported by the Office of Naval Research.

REFERENCES


Khanh T. Nguyen received the B.S. degree in physics and mathematics in 1978, the M.S. degree in mathematics in 1979, and the M.S. and the Ph.D degrees in nuclear science in 1980 and 1983, respectively, all from the University of Michigan, Ann Arbor. His Ph.D. research topic was a stability study of the ELMO Bumpy Torus fusion device.

He then joined the Department of Research and Technology, Naval Surface Warfare Center, White Oak, where he was the Lead Theorist for the charged particle beam propagation experimental program. In 1989, he joined the Washington Office of Mission Research Corporation as a Senior Scientist, and later became the leader of the Electromagnetic Applications Group. At MRC, his research efforts were in the areas of charged particle beam propagation, vacuum electronics, compact accelerator development, x-ray and y-ray simulators, and high-power microwave sources development. Since 1994, when he initiated KN Research, he has been an on-site contractor with the Vacuum Electronics Branch, Naval Research Laboratory. His current research emphasis is on the design and modeling of vacuum electronic devices.

Dr. Nguyen was a recipient of the 1987 NSWC Young Professional of Year award.

Gun-Sik Park received the B.S. degree in physics from Seoul National University, Korea, in 1978 and the M.S. and Ph.D. degrees in physics from the University of Maryland, College Park, in 1989.

He was with the Naval Research Laboratory from 1987 to 1995, where he was a Research Physicist in the Vacuum Electronics Branch, Electronics Science and Technology Division, as an on-site contractor with Omega-F, Inc. Currently he is an Assistant Professor in the physics education department of Seoul National University. His main research area is high-power microwave devices including the fastwave devices such as gyrotrons and peniotrons, and slow wave conventional devices.

Jin Joo Choi photograph and biography not available at the time of publication.

Soo-Yong Park photograph and biography not available at the time of publication.

Robert K. Parker (M'85-SM'93-F'95) received the B.S. degree in physics from Allegheny College, Meadville, PA, in 1964, the M.S. degree in space physics engineering from Air Force Institute of Technology, Wright-Patterson AFB, OH, in 1966, and the Ph.D. in nuclear engineering (plasma physics) from the University of New Mexico, Albuquerque, in 1973.

From 1964 to 1972, he served in the United States Air Force as a Scientific Project Officer. Since 1972, he has been with the Naval Research Laboratory, where he has been actively involved in the processes and techniques of coherent radiation generation in nonneutral plasmas. In 1981, he formed the Vacuum Electronics Branch, Electronic Science and Technology Division. In 1992, the Vacuum Electronics Branch was identified as the DoD center of excellence for research and development in vacuum electronics. He is now Head of the VEB. In addition to his role with respect to internal R&D efforts at NRL, he is the administrator of the Navy exploratory development program in vacuum electronics.

Dr. Parker is the Chairman of the Tri-service Vacuum Electronics Committee. He is the Navy Member of the DoD Advisory Group on Electron Devices, Working Group A: Microwaves. He is a Fellow of the American Physical Society and a member of Sigma Xi.